Journal of Sound and Vibration (1998) **212**(3), 469–479

IDENTIFICATION TECHNIQUES OF STRUCTURE CONNECTION PARAMETERS USING FREQUENCY RESPONSE FUNCTIONS

H. Y. Hwang

University of Ulsan, Ulsan, Korea 680-749

(*Received* 12 *March* 1996, *and in final form* 10 *November* 1997)

The identification method for stiffness constant parameters and damping coefficient parameters of connections using test data is derived when a structure is attached to another structure via connections. Because of the inherent difficulties of deriving spatial models from test data, the frequency response function (FRF) is used as a response model. The identification method using the FRFs works for each discrete frequency so that the connection properties can be found for each frequency and can be averaged using statistical methods for an accurate identification. If highly sensitive regions are excluded, this identification method for connection using FRFs gives accurate estimations and can be applied to a general structure in an easier manner than the modal model methods which require a mathematical model of the mass, damping and stiffness matrices (or natural frequencies and mode shapes).

1. INTRODUCTION

Coupling between one structure and a neighbouring structure via the structural connections which inevitably exist can have a major influence on the actual values of the natural frequencies and mode shapes. Hence, accurate dynamic response of the complex structure is frequently restricted by the capabilities to describe the connection part of the structure properly. For example, the popular finite element method may not produce satisfactory dynamic behaviour of a mechanical system due to the uncertainty of the connection properties. The real structure usually includes many different kinds of joints (bolted joints, sliding joints, socket joints, etc.) which contain at least some amount of non-linearity (e.g., clearance non-linearity, dry friction, localized plasticity, etc.). Since boundary conditions are determined by the connecting joints in a structure, the dynamic characteristics of the structure are greatly influenced by the modelling of connections between structural components. This leads to an increasing need for developing improved analytical models for connection parts.

However, it is extremely difficult to develop accurate analytical models for connection parts by just considering the geometry and properties of a structure. Analytical models for connections may have modelling errors due to incorrect stiffness and mass properties of connections and non-linearities of connections and many other incorrect configurations of connections. Hence, some means of developing confidence in or of validating analytical models of connection parts is needed. One way is to use vibration test data as a criterion, expecting that the analytical modellings of connections agree with test data.

0022–460X/98/180469 + 11 $25.00/0/sv971433

In this paper, the identification method for stiffness constant parameters and damping coefficient parameters of connections using test data is derived when a structure is attached to another structure via connections. In this modified method, frequency response functions are used to find physical connection parameters instead of natural frequencies and mode shapes. By using this new method, connection parameters are directly obtained from the measured receptances and inertances without introducing mathematical models of the mass, damping, and stiffness matrices from test natural frequencies and mode shapes. This research greatly reduces the quantity of test data that must be obtained from the coupled system because it requires only one numerical value of the frequency response function among various frequency range values for each connection. One frequency response function contains an infinite number of numerical values. This is particularly useful when it is not realistic to obtain a complete set of vibration test data of natural frequencies and mode shapes for a coupled structure, such as large space structures, large ocean structures, or complex turbomachinery.

Many attempts at applying frequency-domain test analysis techniques have failed in the past because of inaccuracies in the FRFs used. Recently, with the availability of better data acquisition hardware and techniques, the use of measured FRFs for frequency-domain techniques has become more practical.

First, in this paper, equations to find stiffness and damping constants using the test data of frequency response functions are derived from the equations of motion for a structure with connections. Next, for a better understanding of the derived equations, an example consisting of a continuous beam system is examined.

A number of papers have been published which discuss theoretical structural dynamics, and a considerable amount of experimental research has been performed which investigated coupled structures [1–5]. However, the objective of most of these studies was to estimate the overall behaviour of the coupled structure. Only a few studies have tried to identify the connection parameters themselves using experimental data [1, 2]. Huckelbridge and Lawrence [1, 2] developed a procedure for identifying physical connection properties from free and forced response test data, then verified it by utilizing a system having both a linear and non-linear connection. Interface connections in both the translational and rotational directions were addressed. Connection properties were computed in terms of physical parameters so that the physical characteristics of the connections could be better understood, in addition to providing improved input for the system model. They used the modal model methods which require mass and stiffness matrices from test natural frequencies and mode shapes so that identified results have an error and are highly dependent on the amounts of test data and the mode selected.

Ewins [3, 10] reviewed some of the methods available for making vibration analyses of complex structures where one or more of the component substructures is modelled from experimental, rather than theoretical data. The study concentrated on methods of analysis which were based on response properties.

Time-domain and frequency-domain methods for coupling substructures with general linear damping were considered in reference [4]. The time-domain method used state variable notaion for each substructure. The frequency-domain method utilized the discrete Fourier transform and fast Fourier transform to get transient response solutions. Suarez and Singh [5] presented a method for synthesizing the real modes of substructures to obtain the complex modes of the combined structure including non-proportional damping effects which may exist in the combined structure.

Wang and Liou [6] proposed a method to synthesize the frequency response functions of a structure which is composed of two beams and linear joint springs and dampers. They introduced a simple method based on statistical criteria to overcome the problem caused by measurement noise. In the synthesis process, it was assumed that one substructure was fixed completely. However, in an actual measurement situation, grounding a substructure is a very difficult task. In reference [7], Wang identified the stiffness, mass and damping matrices of a linear mechanical system using a weighted frequency response function combined with the instrumental variable method. The effect of measurement noise was of particular concern.

Yoshimura [8, 9] made experimental measurements to obtain vibratory characteristics and quantitative values of the rigidity and damping at a bolted joint, welded joint and slide way which were the representative joints in the machine tool structure under study. He described methods and procedures for adapting equivalent spring stiffness and damping coefficients to bolted joints and slide ways in computer-aided machine tool design.

Chen and Cherng [11] presented a simple and effective modal synthesis method via combined experimental and finite element techniques in which the "constraint modes method" was used to determine the dynamic properties of complex structures. To satisfy the rotational compatibility at the common boundary, an experimental procedure was proposed to measure the generalized dynamic compliance.

In reference [12], a unique methodology was proposed to identify the joint structural parameters of a machine tool by combining the dynamic data system methodology with the finite element method. The structural parameter identification of a simple system with a complete modal matrix was introduced together with modal analysis by the dynamic data system method.

In reference [13], both time-domain and frequency-domain component mode synthesis methods were reviewed and according to the types of component modes used in synthesis process, the methods were further subdivided into cases such as free-interface component normal modes, constraint modes, inertia-relief attachment modes, etc. A large number of references about component mode synthesis methods are also given in this paper.

Ghlaim and Martin [14] solved a matrix set involving eigenvalues and eigenvectors of the substructure, together with a connection matrix to give the complex roots of the system. Reduction was applied by approximating the substructures by a reduced set of eigenvalues and an equally reduced set of displacements in the eigenvector.

Ewins and Henry [15] provide the necessary introduction and grounding for a study of vibration characteristics of individual turbomachine blades including joint characteristics.

Most of these experimental studies of connection parameters involved comparing the frequency response functions for a range of different cases instead of identifying numerical values of connections [3–15].

In this paper, instead of comparing the frequency response functions with connections, methods to find numerical values of the connection parameters for complicated real structures using frequency response function data have been developed.

2. FORMULATION OF THE EQUATIONS OF MOTION

Assume that a hypothetical structural component is attached to a fixed structure via springs and dampers as shown in Figure 1.

For a structure *with connections*, equations of motions are given by reference [6],

$$[M]\begin{Bmatrix} \ddot{x}_A \\ \ddot{x}_a \end{Bmatrix} + [D]\begin{Bmatrix} \dot{x}_A \\ \dot{x}_a \end{Bmatrix} + [K]\begin{Bmatrix} x_A \\ x_a \end{Bmatrix} = \begin{Bmatrix} \{f_A\} \\ \{f_a\} + \{f_{ja}\} \end{Bmatrix}, \tag{1}$$

where a represents the region on the connection of the structure, A represents the region excluding region a on the structure, $\{f_{ja}\}$ represents the interface force acting on region a, $\{f_A\}$ and $\{f_a\}$ are applied forces.

Interface force $\{f_{ja}\}$ at connections can be expressed as

$$\{f_{ja}\} = -\begin{Bmatrix} k_1\, x_{a1} \\ k_2\, x_{a2} \\ \vdots \\ k_n\, x_{an} \end{Bmatrix} - \begin{Bmatrix} d_1\, \dot{x}_{a1} \\ d_2\, \dot{x}_{a2} \\ \vdots \\ d_n\, \dot{x}_{an} \end{Bmatrix} = -[K_c] \begin{Bmatrix} x_{a1} \\ x_{a2} \\ \vdots \\ x_{an} \end{Bmatrix} - [D_c] \begin{Bmatrix} \dot{x}_{a1} \\ \dot{x}_{a2} \\ \vdots \\ \dot{x}_{an} \end{Bmatrix}, \tag{2}$$

where

$$[K_c] = \begin{bmatrix} k_1 & & & \\ & k_2 & & \\ & & \ddots & \\ & & & k_n \end{bmatrix}, \qquad [D_c] = \begin{bmatrix} d_1 & & & \\ & d_2 & & \\ & & \ddots & \\ & & & d_n \end{bmatrix}. \tag{3}$$

Then, equation (1) can be expressed as

$$[M]\begin{Bmatrix} \ddot{x}_A \\ \ddot{x}_a \end{Bmatrix} + \left([D] + \begin{bmatrix} [0] & [0] \\ [0] & [D_c] \end{bmatrix}\right)\begin{Bmatrix} \dot{x}_A \\ \dot{x}_a \end{Bmatrix} + \left([K] + \begin{bmatrix} [0] & [0] \\ [0] & [K_c] \end{bmatrix}\right)\begin{Bmatrix} x_A \\ x_a \end{Bmatrix} = \begin{Bmatrix} \{f_A\} \\ \{f_a\} \end{Bmatrix}. \tag{4}$$

Let

$$\{q_c\} = \{\{x_A\}, \{x_a\}\}^T, \qquad \{f_c\} = \{\{f_A\}, \{f_a\}\}^T, \qquad [D_{c4}] = \begin{bmatrix} [0] & [0] \\ [0] & [D_c] \end{bmatrix},$$

$$[K_{c4}] = \begin{bmatrix} [0] & [0] \\ [0] & [K_c] \end{bmatrix}. \tag{5}$$

Now, equation (4) can be expressed as

$$[M]\{\ddot{q}_c\} + [D]_w\,\{\dot{q}_c\} + [K]_w\,\{q_c\} = \{\{f_c\}\}, \tag{6}$$

where $[D]_w = [D] + [D_{c4}]$, $[K]_w = [K] + [K_{c4}]$, and subscript w represents connections.

Similarly, for a structure *without connections*, equations of motions are given by

$$[M]\{\ddot{q}_c\} + [D]\{\dot{q}_c\} + [K]\{q_c\} = \{\{f_c\}\}. \tag{7}$$

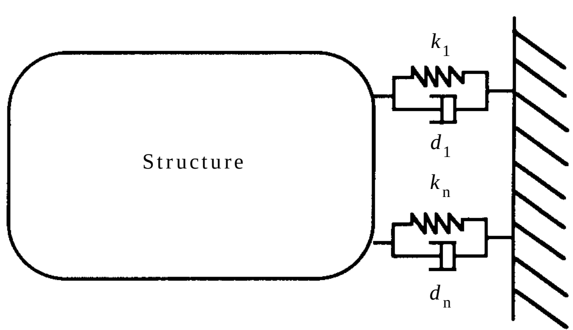

Figure 1. Fixed structure with connections.

Let $\{f_c\} = \{f\}\, e^{j\omega t}$, $\{q_c\} = \{x\}\, e^{j\omega t}$, so that equations (6) and (7) can be written as:

$$([K]_w + j\omega[D]_w - \omega^2[M])\{x\}\, e^{j\omega t} = \{f\}\, e^{j\omega t}, \tag{8}$$

$$([K] + j\omega[D] - \omega^2[M])\{x\}\, e^{j\omega t} = \{f\}\, e^{j\omega t}. \tag{9}$$

Then, using notations in equation (6), connection parameters, $j\omega[D_{c4}] + [K_{c4}]$, can be found using the following equations.

$$j\omega[D_{c4}] + [K_{c4}] = j\omega[D]_w - j\omega[D] + [K]_w - [K]. \tag{10}$$

By adding and subtracting $-\omega^2[M]$ to the left-hand-side, equation (10) becomes

$$j\omega[D_{c4}] + [K_{c4}] = ([K]_w + j\omega[D]_w - \omega^2[M]) - ([K] + j\omega[D] - \omega^2[M]), \tag{11}$$

where subscript w represents the structure with connections.

From equation (9), frequency response function, $[H(\omega)]$, can be represented by

$$[H(\omega)] = \{x\}/\{f\} = ([K] + j\omega[D] - \omega^2[M])^{-1}. \tag{12}$$

A frequency response function is the relation between the Fourier transform of the system output (response) and the Fourier transform of the system input (applied force). Hence, equation (11) becomes

$$j\omega[D_{c4}] + [K_{c4}] = [H]_w^{-1} - [H]_{w/0}^{-1}. \tag{13}$$

Alternatively,

$$j\omega[D_{c4}] + [K_{c4}] = j\omega \begin{bmatrix} [0] & [0] \\ [0] & \begin{bmatrix} d_1 & & & \\ & d_2 & & \\ & & \ddots & \\ & & & d_n \end{bmatrix} \end{bmatrix} + \begin{bmatrix} [0] & [0] \\ [0] & \begin{bmatrix} k_1 & & & \\ & k_2 & & \\ & & \ddots & \\ & & & k_n \end{bmatrix} \end{bmatrix} = [H]_w^{-1} - [H]_{w/o}^{-1}. \tag{14}$$

Thus, damping coefficient d_i and spring constant k_i can be determined by measuring FRF of the structure with and without connections.

3. EXAMPLE: IDENTIFICATION OF CONNECTIONS USING FRFs

In this section, the identification of connections using frequency response functions (FRFs) will be carried out for a continuous beam system. For a system model shown in Figure 2(a), frequency response functions for the connected structure and unconnected structure could be obtained from an experiment with the physical model. However, representative FRFs that could be obtained from the test will be synthesized numerically from the continuous system model given in Figure 2(b).

The finite element model of a cylinder type beam in Figure 2(b) was used to create natural frequencies and modes shapes. The outer diameter and the inner diameter of the cylinder type beam are 2·00 and 1·75 in. each and the length of the beam is 75 in. Young's

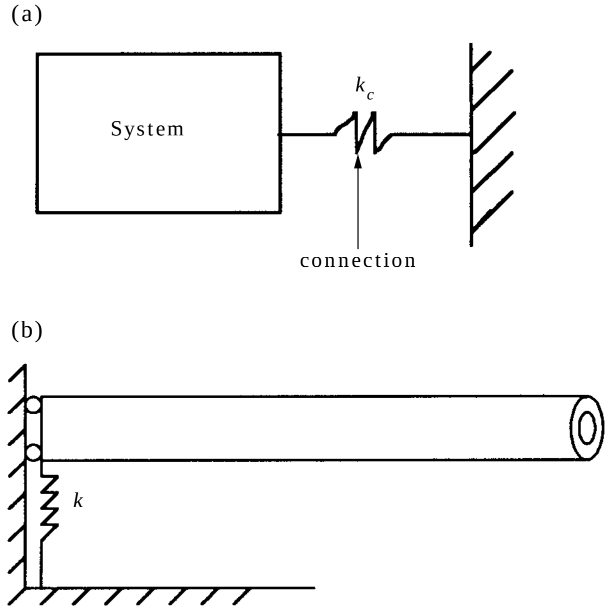

Figure 2. Example of a beam system for identification of connection using FRFs: (a) system model; (b) beam model for frequency response functions.

modulus is 30×10^6 psi and mass density is $5{\cdot}2827 \times 10^{-4}$ (lb — s^2/in.^{2}). The first four natural frequencies of the beam with free–free boundary conditions from experiments are given by 84·0, 232·0, 450·0 and 734·0, excluding rigid body modes. Analytical natural frequencies from the finite element model are given by 85·3, 234·0, 456·4, 750·6, and so on.

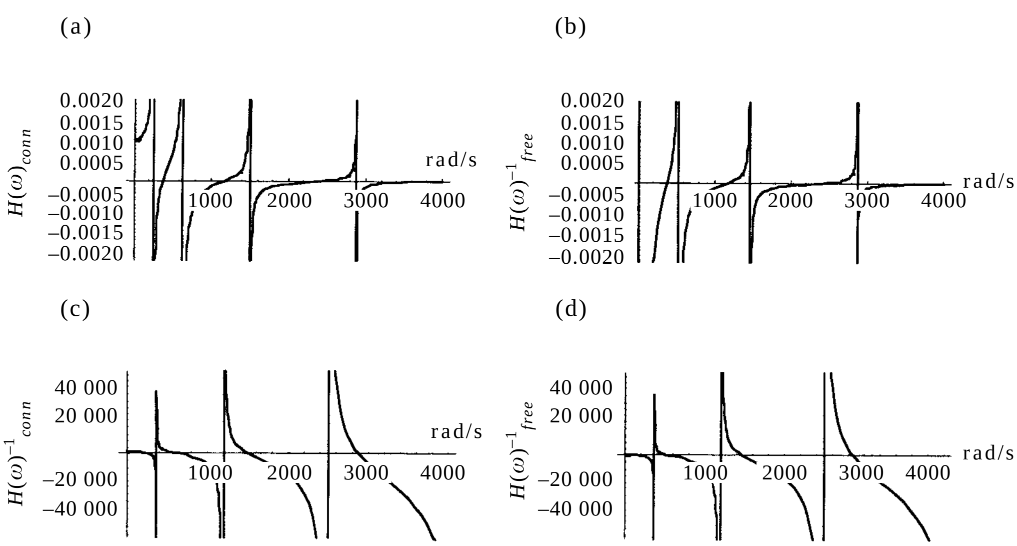

Figure 3. Frequency response functions and inverse frequency response functions for a beam system model: (a) with connection ($H(\omega)_{conn}$); (b) without connection ($H(\omega)_{free}$); (c) with connection ($H(\omega)^{-1}_{conn}$); (d) without connection ($H(\omega)^{-1}_{free}$).

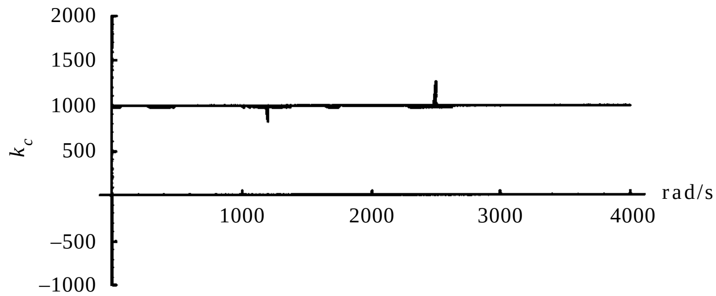

Figure 4. Identified connection spring constant, k_c, as frequency varies.

Frequency response functions and inverse frequency response corresponding to the given models with connection stiffness 10^3 (lb/in.) can be numerically simulated using the following frequency response functions and are plotted in Figure 3.

$$\begin{aligned} H(\omega)_{conn} = {} & 0{\cdot}000000^2/(0{\cdot}00001^2 - \omega^2) + 6{\cdot}342734^2/(257{\cdot}7124^2 - \omega^2) \\ & + 11{\cdot}51112^2/(638{\cdot}2883^2 - \omega^2) \\ & + 10{\cdot}57203^2/(1505{\cdot}402^2 - \omega^2) + 9{\cdot}988351^2/(2884{\cdot}47^2 - \omega^2) \\ & + 9{\cdot}628159^2/(4725{\cdot}742^2 - \omega^2) \\ & + 9{\cdot}292601^2/(7015{\cdot}026^2 - \omega^2) + 8{\cdot}921628^2/(9743{\cdot}036^2 - \omega^2) \\ & + 8{\cdot}492523^2/(12901{\cdot}36^2 - \omega^2) \\ & + 7{\cdot}99677^2/(16480{\cdot}76^2 - \omega^2) + 7{\cdot}434394^2/(20469{\cdot}23^2 - \omega^2) + \cdots, \end{aligned}$$

$$\begin{aligned} H(\omega)_{free} = {} & 3{\cdot}980242^2/(0{\cdot}00001^2 - \omega^2) + 9{\cdot}20543^2/(0{\cdot}00001^2 - \omega^2) \\ & + 9{\cdot}964013^2/(536{\cdot}2135^2 - \omega^2) \\ & + 9{\cdot}851579^2/(1470{\cdot}308^2 - \omega^2) + 9{\cdot}694061^2/(2867{\cdot}631^2 - \omega^2) \\ & + 9{\cdot}482751^2/(4716{\cdot}072^2 - \omega^2) \\ & + 9{\cdot}210876^2/(7008{\cdot}923^2 - \omega^2) + 8{\cdot}871694^2/(9738{\cdot}974^2 - \omega^2) \\ & + 8{\cdot}460328^2/(12898{\cdot}58^2 - \omega^2) \\ & + 7{\cdot}975309^2/(16478{\cdot}82^2 - \omega^2) + 7{\cdot}419817^2/(20467{\cdot}88^2 - \omega^2) + \cdots, \end{aligned}$$

In simulating FRF of a beam with free–free boundary conditions, rigid body modes were added that are the first and the second terms in the above second equation. To avoid numerical difficulties in the inverse process, the natural frequency of the rigid body mode was assumed to be 0·00001 (rad/s) and an arbitrary rigid body mode shape can be chosen.

If it is now assumed that the above computed frequency response functions are those obtained from a test, using equation (14), the connection spring constant, k_c, can be found using the following equation.

$$k_c = H(\omega)^{-1}_{conn} - H(\omega)^{-1}_{free}. \tag{15}$$

It should also be noted that this expression is not constant but is frequency-dependent unlike methods involving eigenvalues and modal matrices.

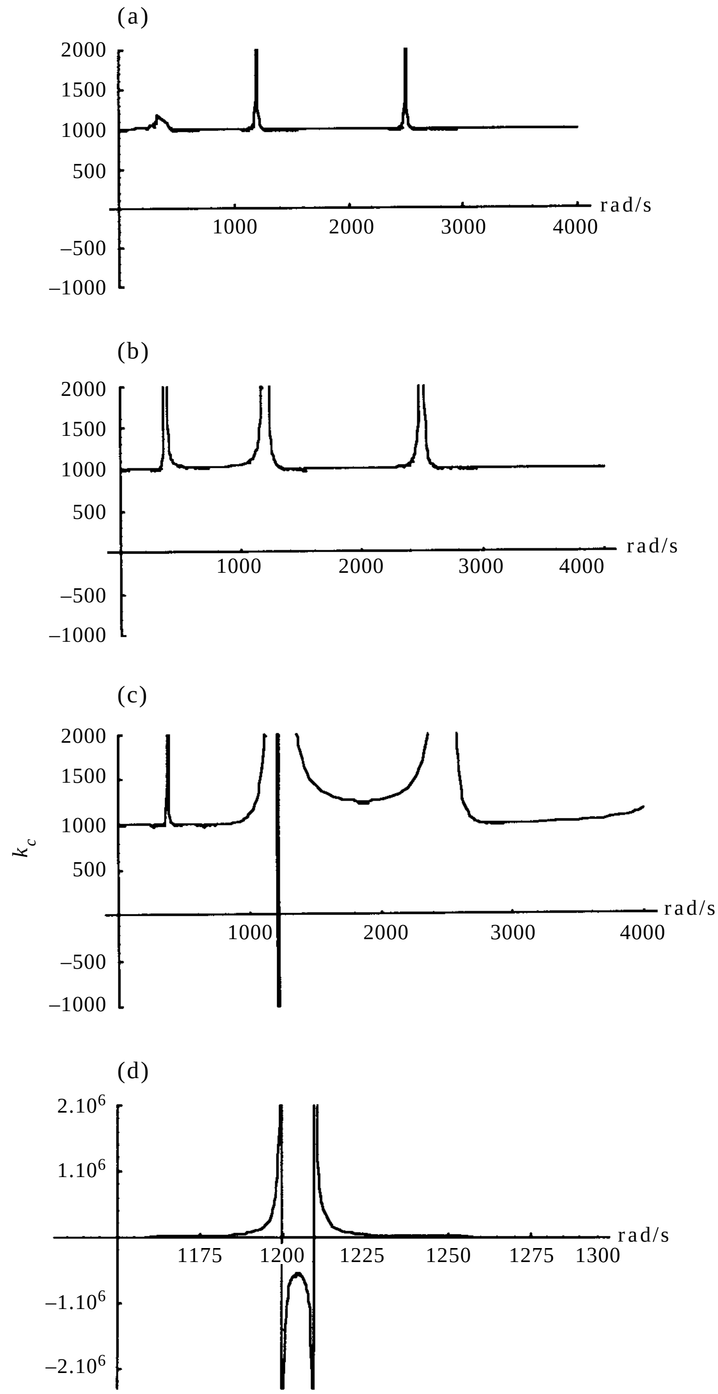

Figure 5. Identified connection spring constant, k_c, when measurement errors exist in the resonance frequencies: (a) 1% error in the first non-rigid body resonant frequency of $H(\omega)_{conn}$; (b) 1% error in the second non-rigid body resonant frequency of $H(\omega)_{conn}$; (c) 1% error in the third non-rigid body resonant frequency of $H(\omega)_{conn}$; (d) expanded plot of data in (c).

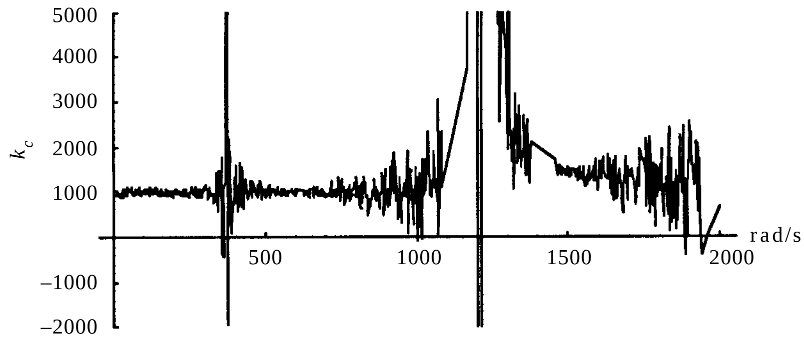

Figure 6. Identified connection spring constant, k_c, for uniformly distributed 10% random noises in magnitudes of $H(\omega)_{conn}$ and $H(\omega)_{free}$.

Using equation (15), the identified connection spring constant, k_c, is plotted in Figure 4 as a function of frequency. Except for small peaks, the identified connection spring constant was uniformly 10^3 (lb/in.) (the assumed value). The small peaks occurred at the frequency where the FRF becomes zero and it is the result of finite numerical precision in the computation. In practice it may be necessary to use some type of statistically averaging process for excluding these kinds of peaks over a range of frequencies to accurately estimate k_c using this approach. Namely, small numerical values of FRFs close to zero should be excluded to avoid infinite values in the inverse process.

In a practical application, the measurements will include errors arising from many potential sources, and these will affect the estimation of k_c. Two different situations will be considered: (1) errors in measurement of the resonant frequencies; (2) errors in the amplitude measurement of the FRFs.

In Figure 5(a), k_c is plotted for the frequency range of 0–4000 rad/s when a 1% error exists in the first non-rigid body resonant frequency. In Figure 5(b), k_c is plotted for the frequency range of 0–4000 rad/s when a 1% error exists in the second non-rigid body resonant frequency. In Figure 5(c), k_c is plotted for the frequency range of 0–4000 rad/s when a 1% error exists in the third non-rigid body resonant frequency. In Figure 5(d), k_c is plotted for the frequency range of 1100–1300 rad/s when a 1% error exists in the third resonant frequency to check double peaks that exist in Figure 5(c). These kind of double peaks can be explained using Figures 3(c) and (d). Since k_c is the difference between these two inverse FRF graphs, a small shift in one peak creates double peaks in the k_c graph.

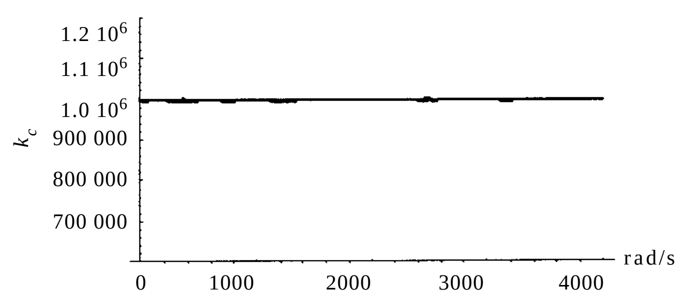

Figure 7. Identified connection spring constant, k_c, for assumed value of $k_c = 10^6$.

From Figure 5, it can be said that the identification of k_c using equation (15) gives accurate results except values near the frequencies where inverse FRFs become infinity.

Next, uniformly distributed random numerical noise was used to simulate measurement errors in the amplitude of the FRFs. At each computed point of the simulated FRF, the amplitude was assumed to be "contaminated" by measurement noise. The noise was simulated by adding a $\pm 10\%$ uniformly distributed random number. Specifically, it was assumed that:

$$k_c = [A_1\, H(\omega)_{conn}]^{-1} - [A_2\, H(\omega)_{free}]^{-1}, \tag{16}$$

where $A_1 = (1 + \alpha)$ and $A_2 = (1 + \alpha)$ and α is a uniformly distributed random number with $|\alpha| \leqslant 10\%$. The resulting plot of k_c as a function of frequency is shown in Figure 6.

If the results in Figure 6 are considered with FRFs or inverse FRFs in Figure 3, it can be shown that a greater error in k_c will be created if the values of FRFs are close to zero (or if the inverse values of FRFs are close to infinity).

For very high connection stiffness cases of $k = 10^6$ (lb/in.), identified spring constants are plotted in Figure 7. As expected, they show slightly higher estimation but still give useful values.

4. CONCLUSIONS

When an analytical model does not exist, experimentally obtained modal parameters must be used for the identification of connection parameters and for the explanation of dynamic properties that are not obvious from the analytical models. However, the cost to measure all possible modes will be outside the budget constraint, and it is likely that modes will be missed or that some of the identified modes are not true modes of the structure. Hence, the modal mode method which uses the natural frequencies and mode shapes cannot be used when it is difficult to create an accurate modal data from the test. The identification method of connections in this paper, which used the FRFs directly, has a better technical merit in this aspect than methods using modal data of natural frequencies and mode shapes [1, 2] to create system matrices M, C, K because FRFs are easily measured dynamic properties of the structure. Besides, many difficulties and restrictions in the calculation of these matrices from test data can be avoided.

The identification method which uses the FRFs, as discussed in the previous section, works for each discrete frequency so that the connection properties can be found for each frequency and can be averaged using statistical methods to overcome the problems caused by measurement noise. The identified connection properties near the frequency ranges where the FRFs are close to zero should be excluded in the averaging process. If those highly sensitive regions are excluded, the identification method for connection using FRFs can be applied to a general structure in an easier manner than the modal model methods which require a full mathematical model of the mass, damping and stiffness matrices (or natural frequencies and mode shapes).

Since highly sensitive regions can be excluded during the averaging process, identified results with only one set of FRFs are more accurate than the modal model method. Identification of connections using modal model methods was accurate only when full modal data was available [16]. Also, if FRFs are directly used to find the connection parameters, curve fitting processes that are required to find natural frequencies and mode shapes can be omitted in obtaining the test data.

This connection identification method of using FRFs can also be applied to the model update methods and damage detections of structures using vibration test data.

REFERENCES

1. A. A. Huckelbridge and C. Lawrence 1987 *Proceedings of the* 1987 *ASME Design Technology Conferences*—11*th Biennial Conference on Mechanical Vibration and Noise*, *Boston*, *MA*. Identification of structural interface characteristics using component mode synthesis.
2. A. A. Huckelbridge and C. Lawrence 1989 *Proceedings of the* 1989 *ASME Design Technology Conferences*—12*th Biennial Conference on Mechanical Vibration and Noise*, *Montreal*, *Quebec*, *Canada*. Characterization of structural connection using free and forced response test data.
3. D. J. Ewins 1985 *Proceedings of the Joint ASCE/ASME Mechanics Conference*, *Albuquerque*, *NM*. Modal test requirements for coupled structure analysis using experimentally derived component models.
4. R. R. Craig, Jr, R. C. Bachmeyer and T. G. Howsman 1984 *Proceedings of the* 4*th International Conference on Applied Numerical Modeling*, *National Cheng-Kung University*, *Tainan*, *Taiwan*, 172–177. Some approaches to substructure coupling with damping.
5. L. E. Suarez and M. P. Singh 1987 *Proceedings of Dynamics of Structure Sessions at Structures Congress*, *Orlando*, *FL*. A modal coupling procedure for structural dynamics.
6. J. H. Wang and C. M. Liou 1989 *Proceedings of* 1989 *ASME Design Technology Conference*, 12*th Biennial Conference on Mechanical Vibration and Noise*, *Montreal*, *Quebec*, *Canada*, 29–36. Experimental substructure synthesis with linear joints.
7. J. H. Wang 1988 *Journal of Sound and Vibration* **125**, 151–167. Mechanical parameters identification with special consideration of noise effects.
8. M. Yoshimura 1977 *Annals of the CIRP* **25**, 193–198. Measurement of dynamic rigidity and damping property for simplified joint models and simulation computer.
9. M. Yoshimura 1979 *Annals of the CIRP* **28**, 241–246. Computer-aided design improvement of machine tool structure incorporating joint dynamic data.
10. D. J. Ewins and M. G. Sainsburd 1972 *Shock and Vibration Bulletin*, **2**, 105–122. Mobility measurements for the vibration analysis of connected structures.
11. W. H. Chen and J. S. Cherng 1985 *Journal of Sound and Vibration* **103**, 1–11. Modal synthesis via combined experimental and finite element techniques with consideration of rotational effects.
12. J. X. Yuan and X. M. Wu 1985 *ASME Journal of Engineering for Industry* **107**, 64–69. Identification of the joint structural parameters of machine tool by DDS and FEM.
13. R. R. Craig, Jr 1985 *Proceedings of the Joint ASCE/ASME Mechanics Conference*, *Albuquerque*, *NM*. A review of time-domain and frequency-domain component mode synthesis method.
14. K. H. Ghlaim and K. F. Martin 1984 *Proceedings of the* 2*nd International Modal Analysis Conference*, **II**, 683–689. Reduced component modes in a damped system.
15. D. J. Ewins and R. Henry *AGARDograph No*. 298, *AGARD Manual on Aeroelasticity in Axial-Flow Turbomachines* **2**, 14-1–14-27. Structural dynamic characteristics of individual blades.
16. H. Y. Hwang 1993 *Doctoral Thesis*, *Georgia Institute of Technology*, *Atlanta*, *GA*. Identification methods for physical parameters in structure.